

 POLITECNICO DI MILANO



**“Matematica e Realtà”, Benevento 10/10/2014**

**Matematica e catastrofi naturali**

Edie Miglio, MOX, Politecnico di Milano.



# Motivazioni





# Motivazioni





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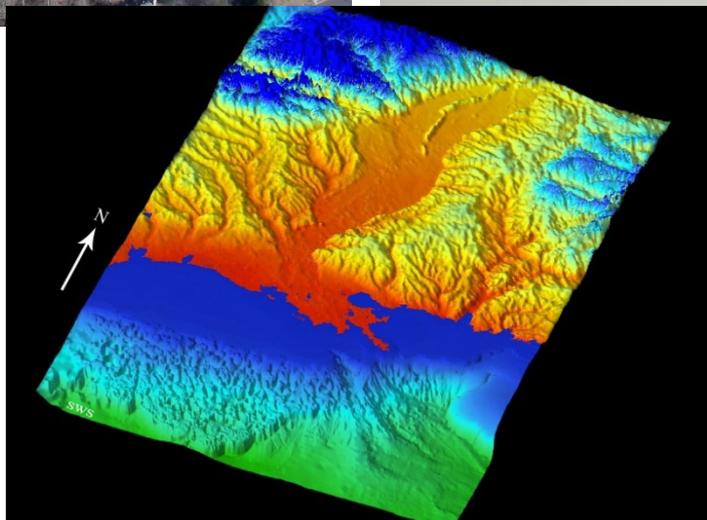


# Motivazioni



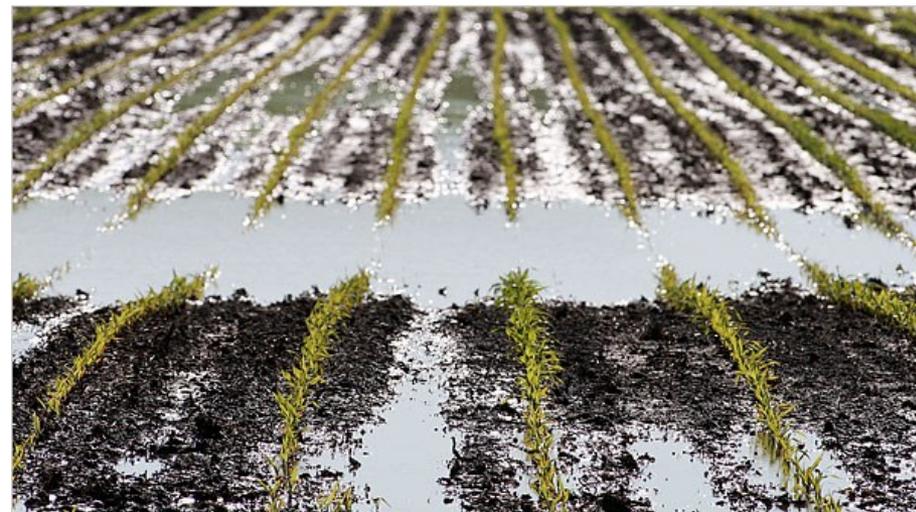


# Motivazioni





## Danni urbani



## Danni rurali



Tre componenti principali:

1. i **modelli matematico/numerici**
2. i **computer (eventualmente paralleli)**
3. le **infrastrutture (di comunicazione, di raccolta dati, ecc.ecc.)**

A noi interesserà mettere in evidenza i primi due punti.



CS: uso di tecniche di calcolo avanzato per capire e risolvere problemi complessi.

Fonde tre elementi:

1. **Algoritmi e software** per risolvere problemi delle scienze ingegneristiche od umane.
2. **Computer e information science** che sviluppano ed ottimizzano sistemi avanzati di hardware, software, networking e data management.
3. Le **infrastrutture di calcolo**.



# Il computer più potente al mondo: 1985 vs 2012

1985: CRAY-2  
4 processori  
1.95 Gflops di picco

(1 Gflops =  $2^{30}$  operazioni al secondo  
 $\approx 10^9$  operazioni al secondo)



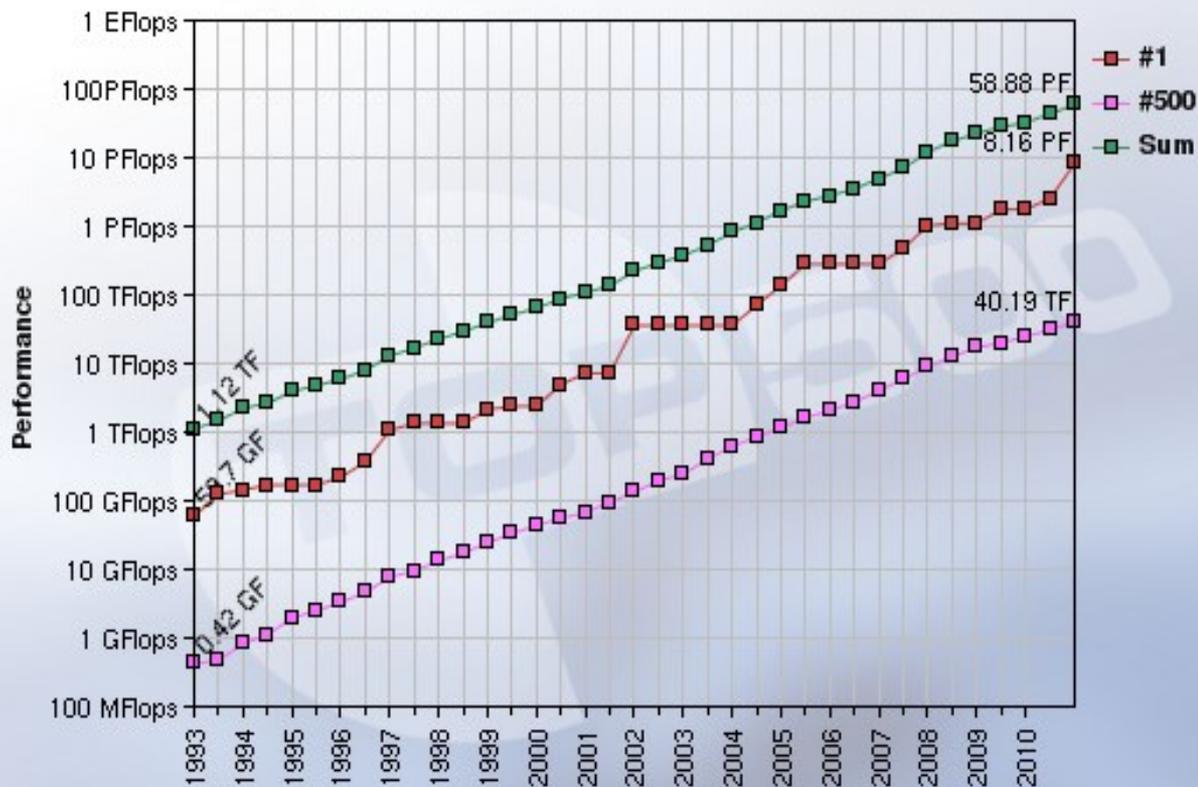
2011: K COMPUTER  
RIKEN Advanced Institute  
for Computational Sciences  
548352 processori  
8.77363e+06 Gflops di picco



# 18 anni di supercomputer: 1993-2012



## Performance Development



16/06/2011

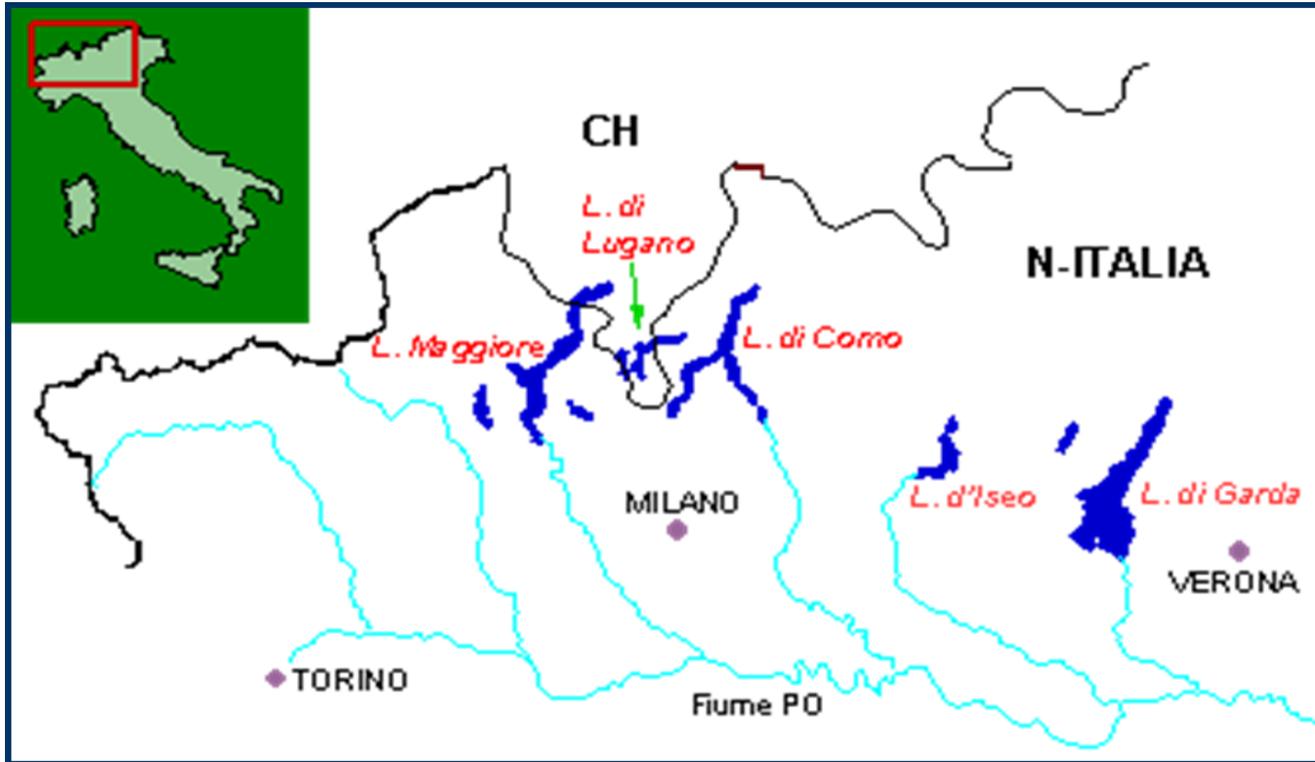
<http://www.top500.org/>

### Prestazioni dei computer

Nome	FLOPS
yottaFLOPS	$10^{24}$
zettaFLOPS	$10^{21}$
exaFLOPS	$10^{18}$
petaFLOPS	$10^{15}$
teraFLOPS	$10^{12}$
gigaFLOPS	$10^9$
megaFLOPS	$10^6$
kiloFLOPS	$10^3$
FLOPS	1



# Un sistema complesso





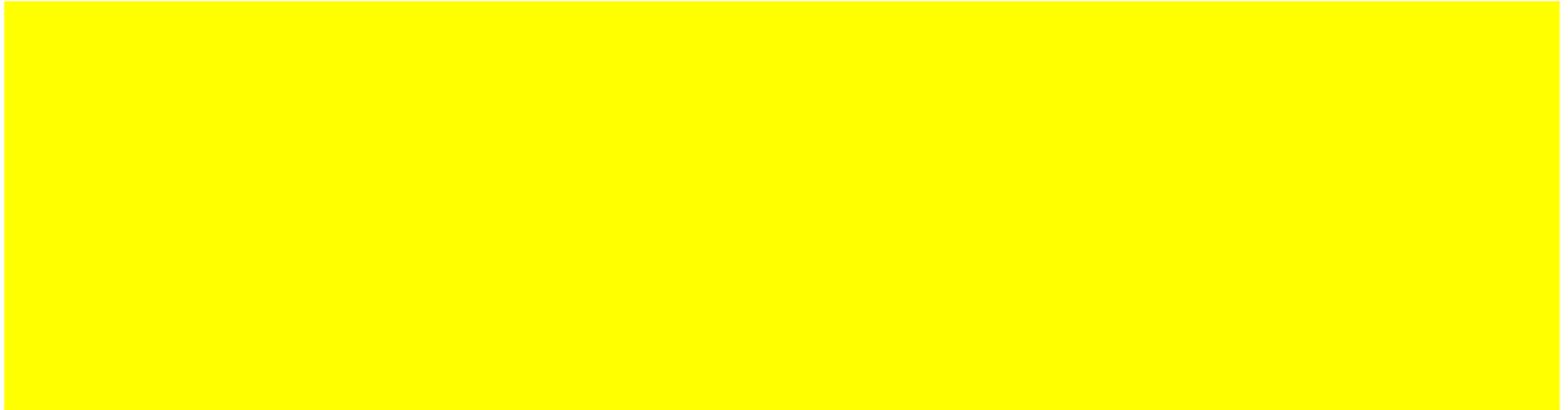
# ONDE DI PIENA, INONDAZIONI E TSUNAMI

Simulazione del movimento delle acque

- Modelli 1D
- Modelli 2D
- Modelli 3D



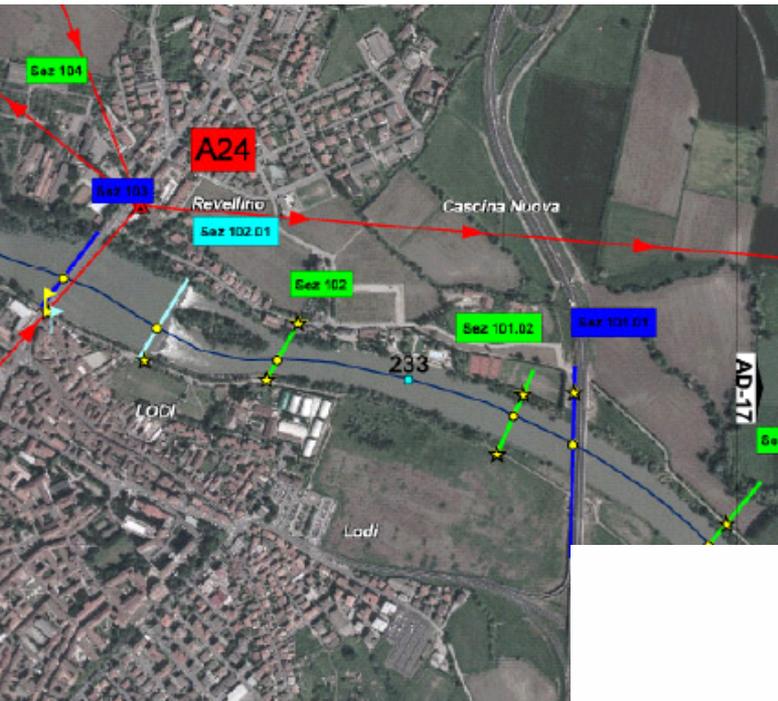
## The 1D open channel equations



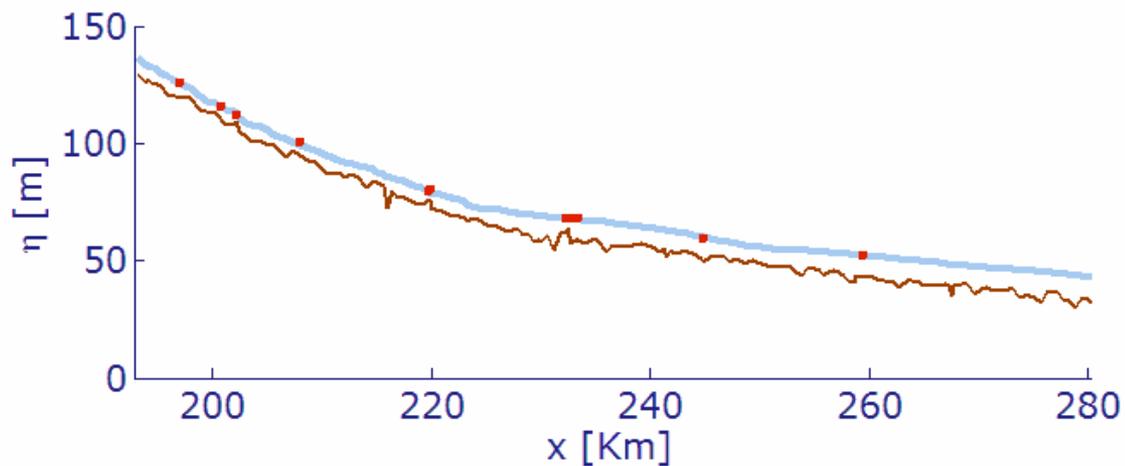
- $A$  : area of the wet cross-section
- $Q(=Au)$ : water discharge
- $h$  total water depth



# Modello 1D di un fiume

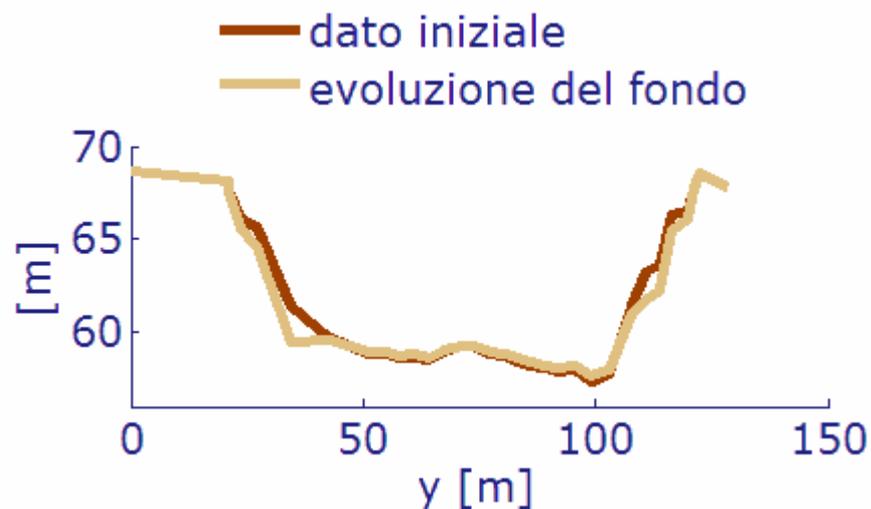
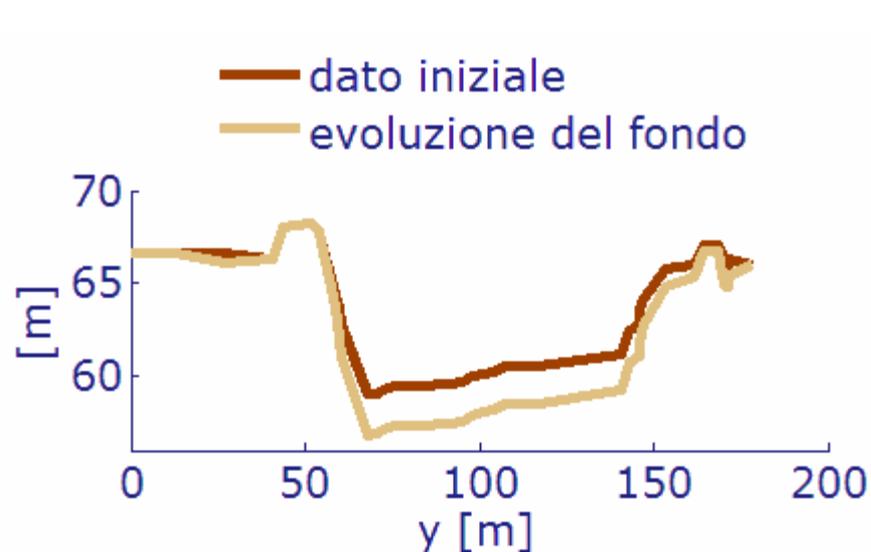
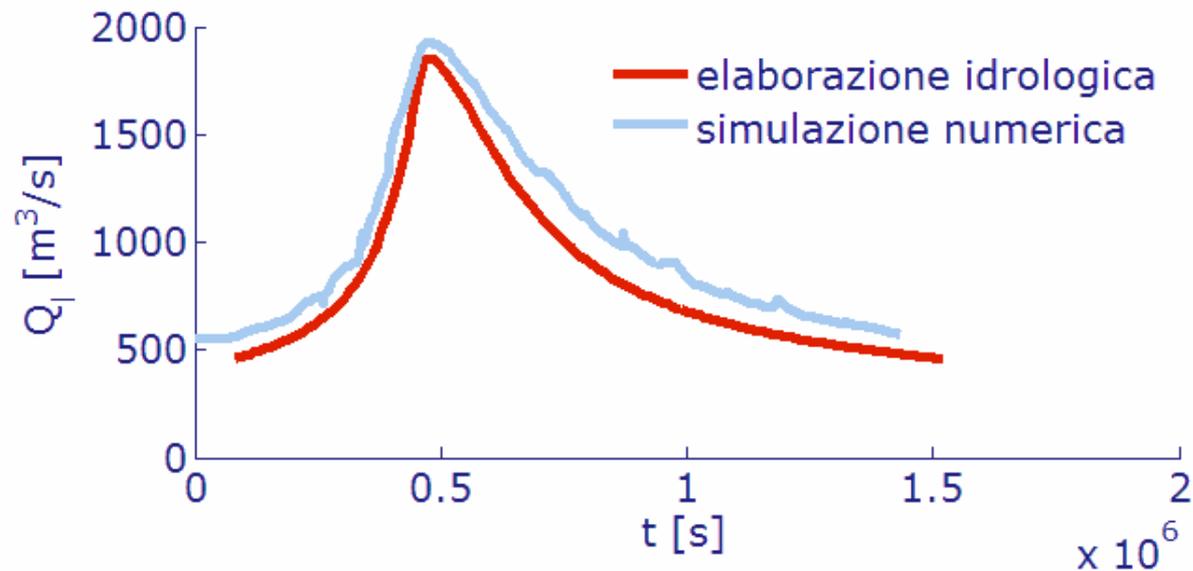


- risultati numerici
- quota del fondo
- quote massime raggiunte durante la piena



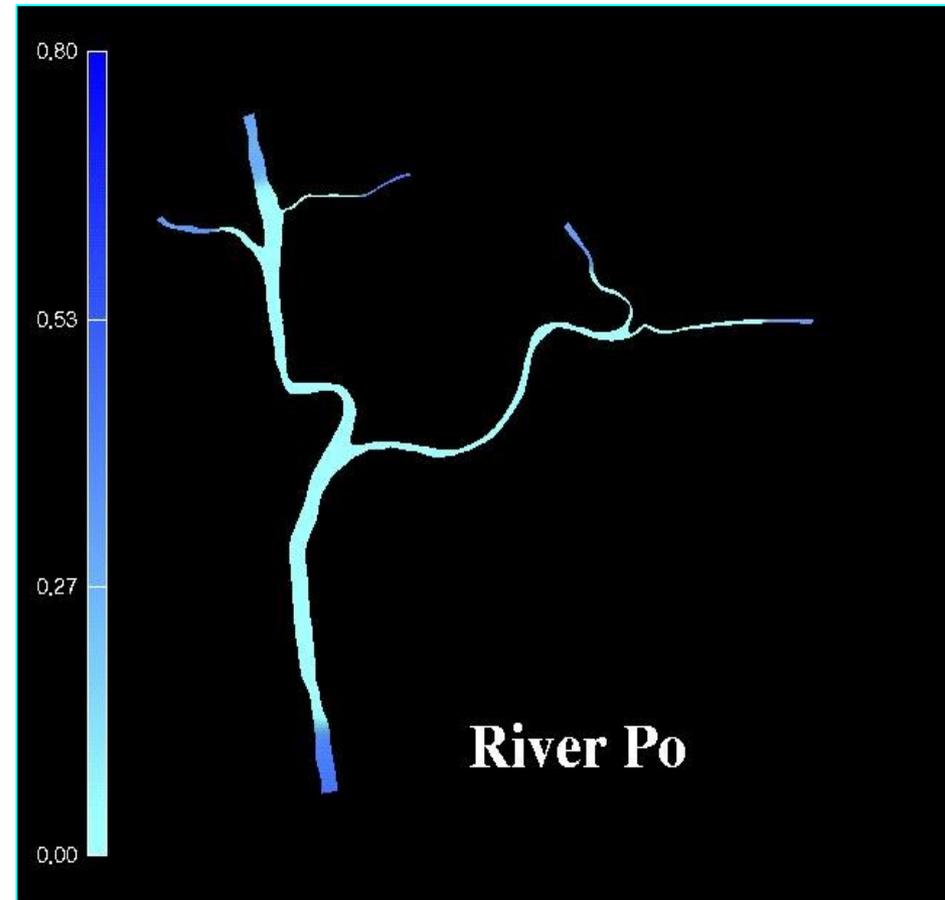


# Modello 1D di un fiume





# Il fiume Po



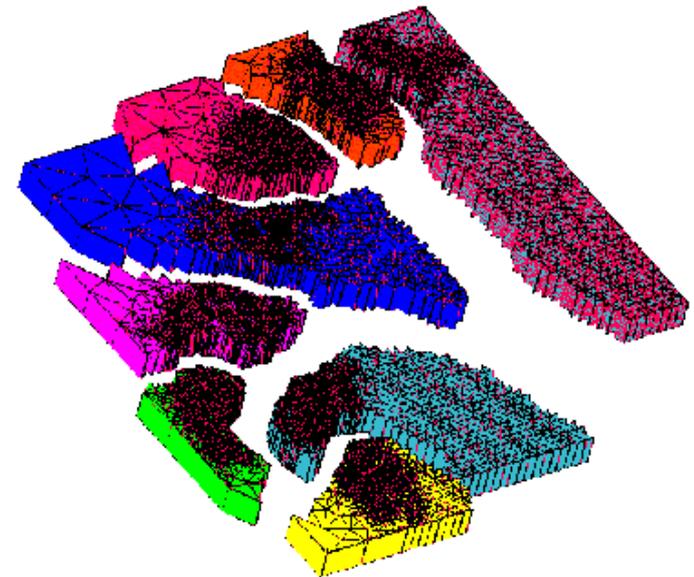


# Simulazione su larga scala: la laguna di Venezia

- 1) Ricostruzione geometrica della batimetria
- 2) Creazione della mesh di calcolo



Simulazione dell'andamento della marea

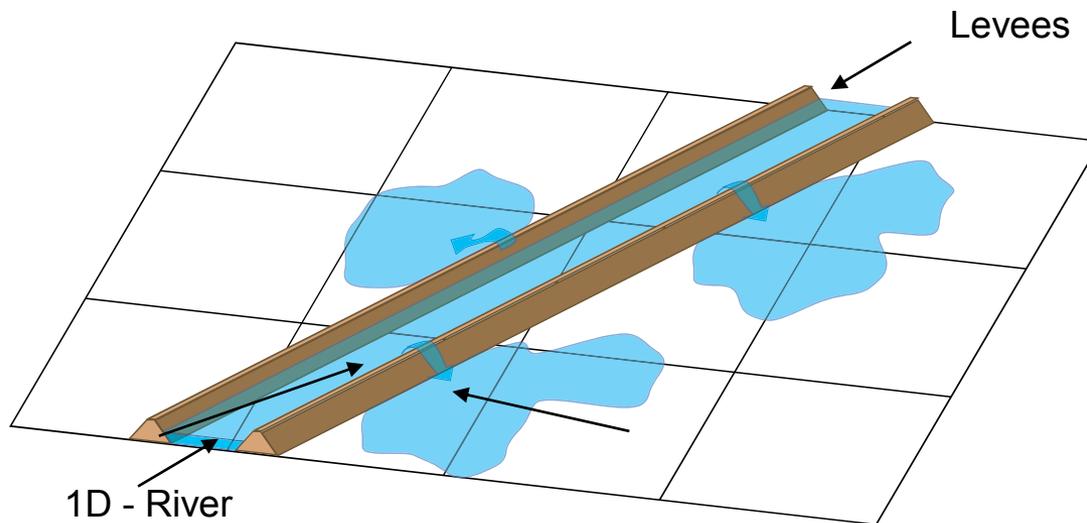


Diffusione e trasporto di un inquinante



# Coupling 1D and 2D Models

Coupling of 1D and 2D models using a special version of Immersed (Cut-Cell) Boundary Method to represent River Flow



Overflow discharge during flood

Overflow discharge From 1D River model is spreading in 2D model

Water overflowing from 1D river will be allowed to spread in 2D model and, reciprocally, water from 2D model can flow into 1D river

Streams and rivers that cannot be captured at the resolution level of DEM constitute a special class of linear feature. A special version of cut-cell boundary method will also be used to represent rivers defined by surveyed cross sections at selected locations.

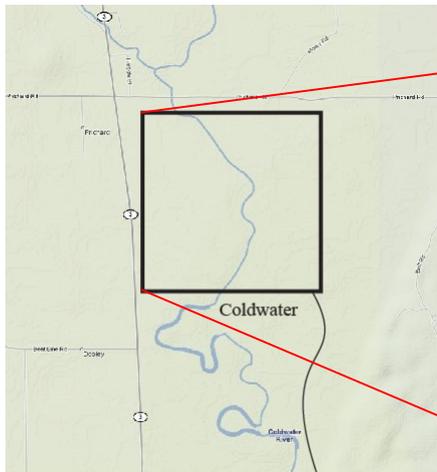
Within the stream, one-dimensional version of SWE is solved using a similar finite volume method.

1D river model and 2D flood model are connected to each other via exchanged discharges.

Exchanged discharge are calculated using weir equation and the flow depths and velocities in 1D and 2D models.



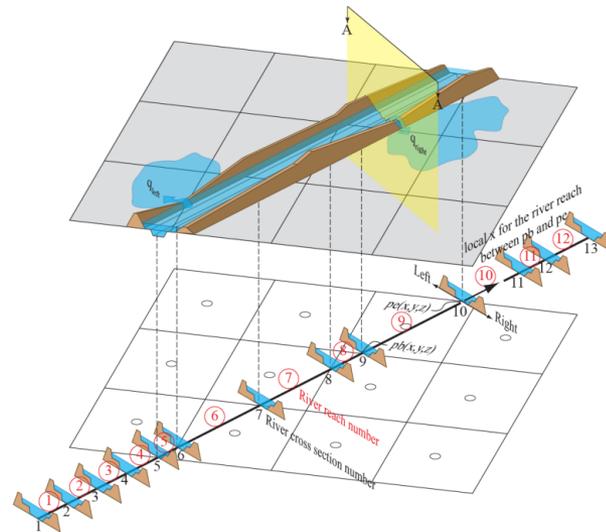
# Coupling 1D and 2D Models



Limits of 2D model



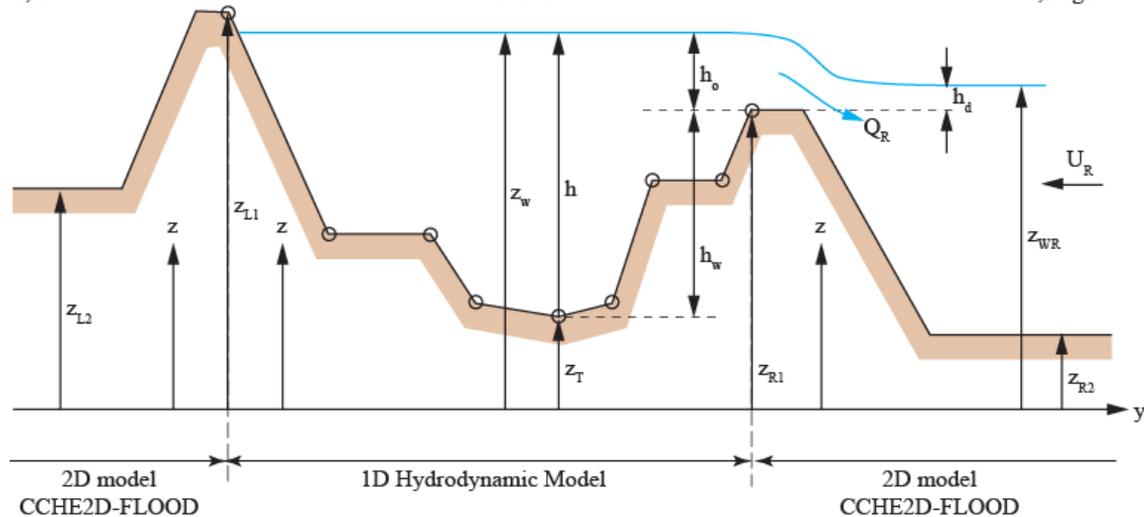
L, Left



Cross Section A-A

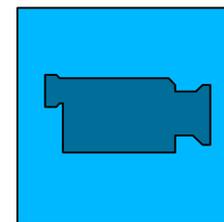
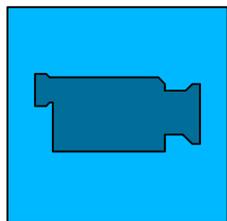
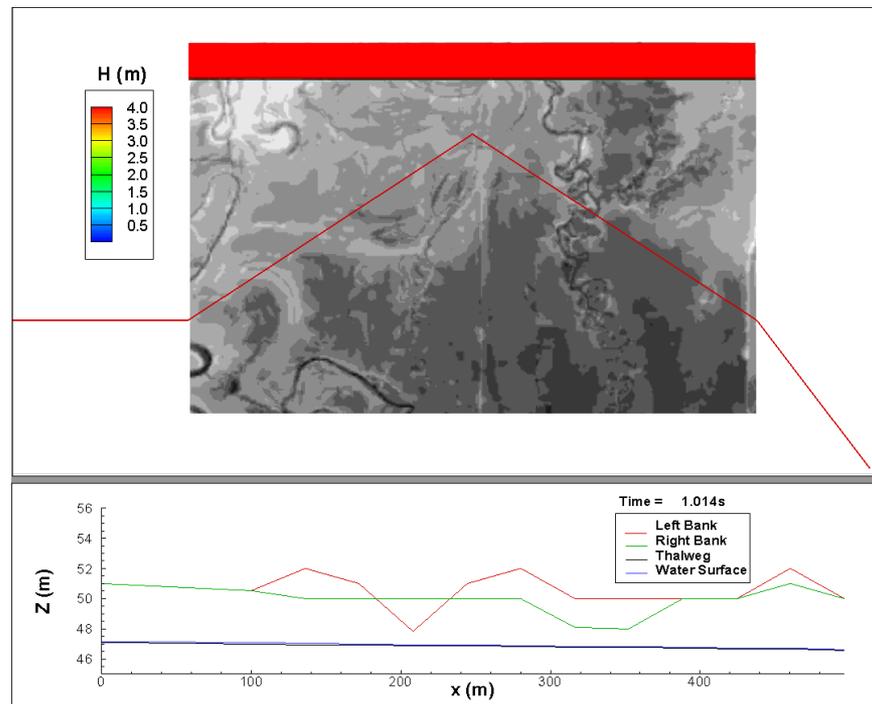
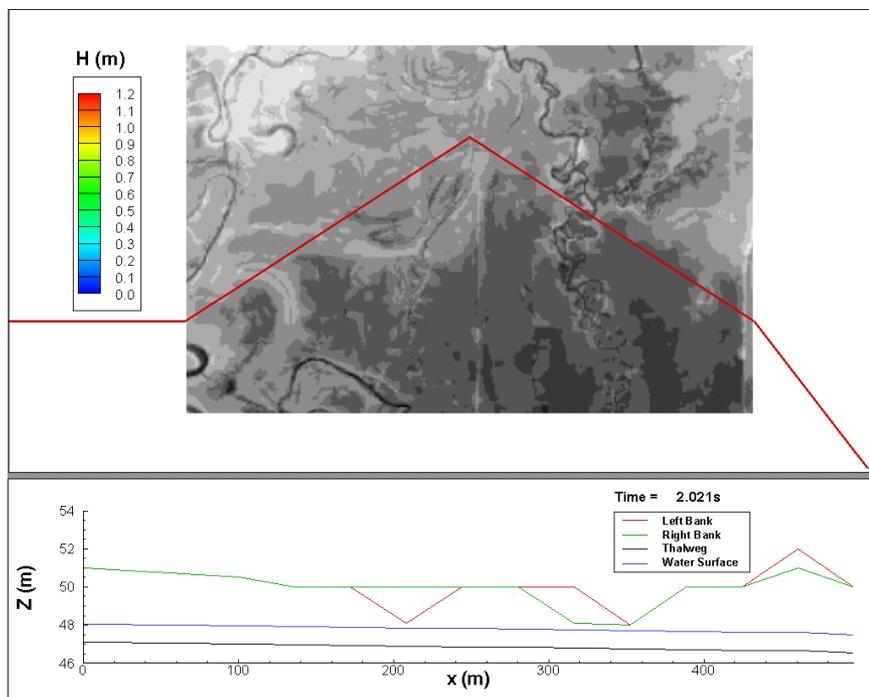
R, Right

Computation of exchange discharges between 1D and 2D models using weir equation. The computation takes into account water depths and velocities in 1D and 2D models.



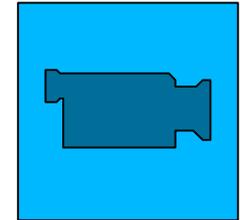
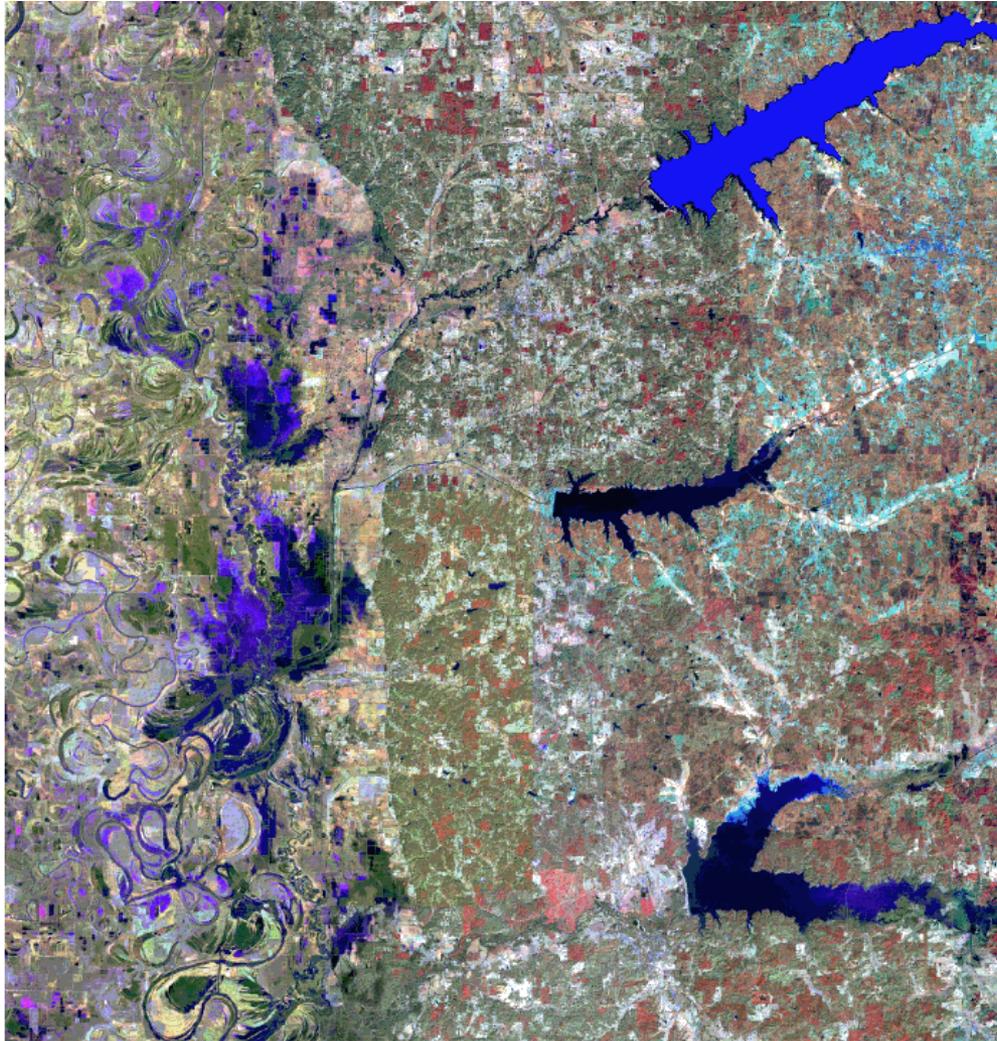


# Simulazione di inondazioni: modelli 1D e 2D in opera contemporaneamente



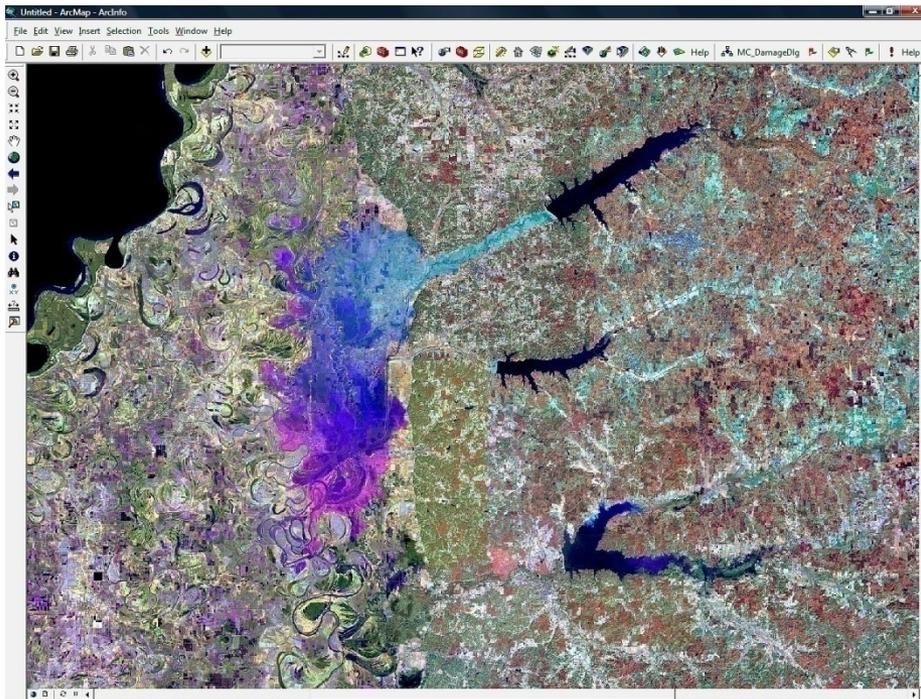


# Rottura di una diga di un bacino artificiale (simulazione di 72h)

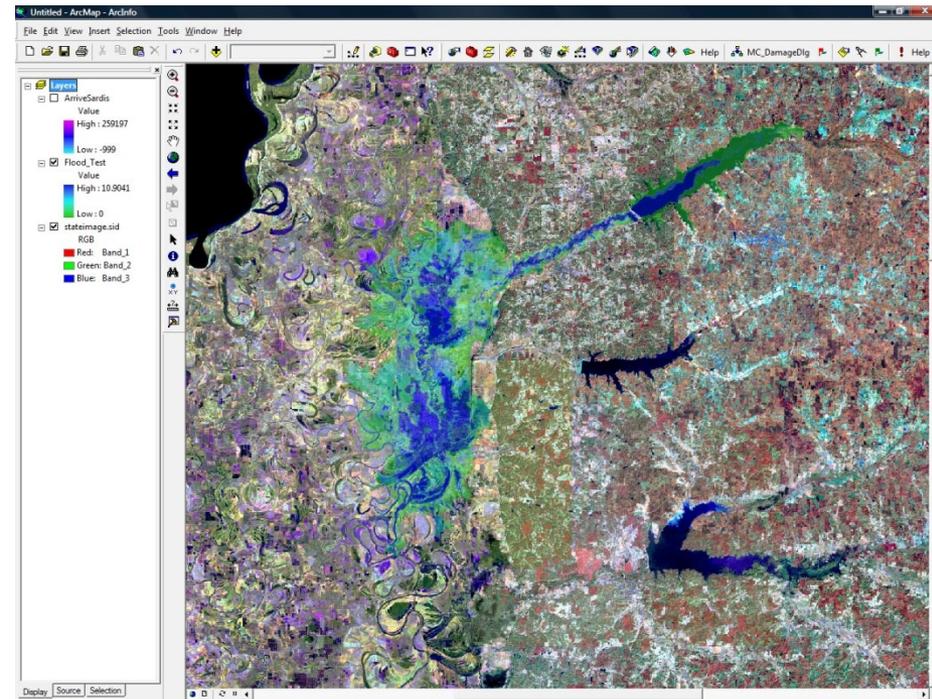




# Decision Support System



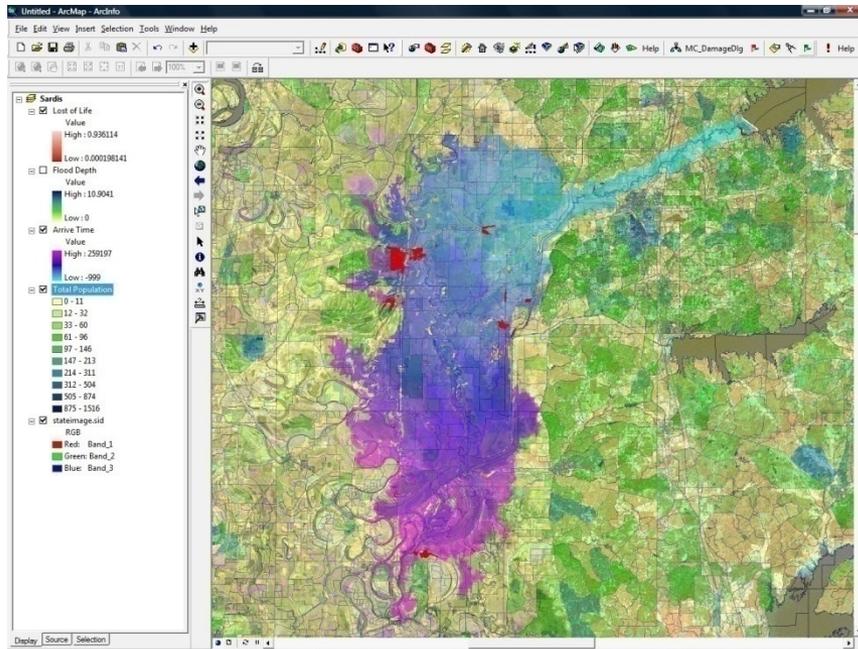
Tempi di arrivo della piena



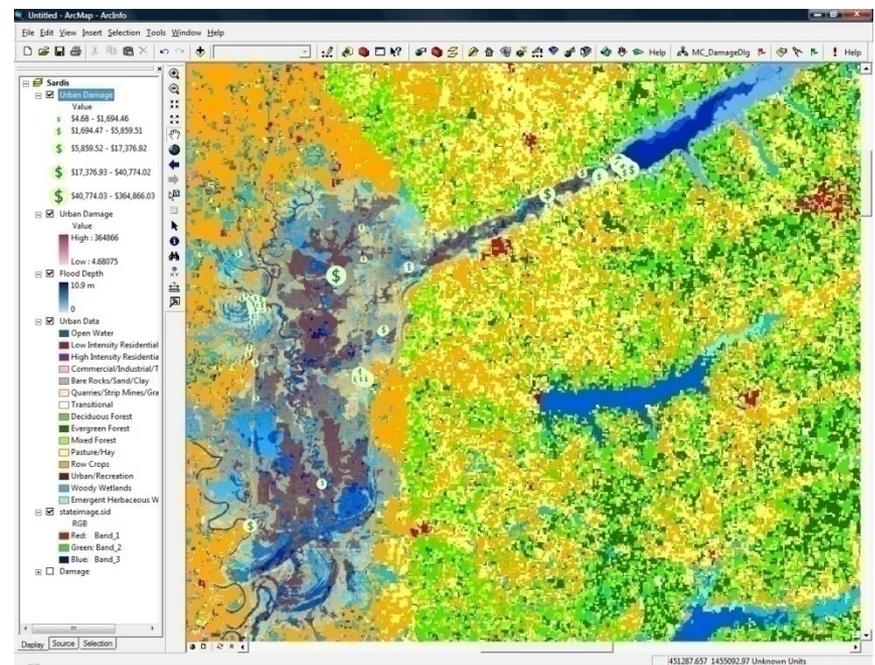
Mappa delle profondità della piena



# Decision Support System



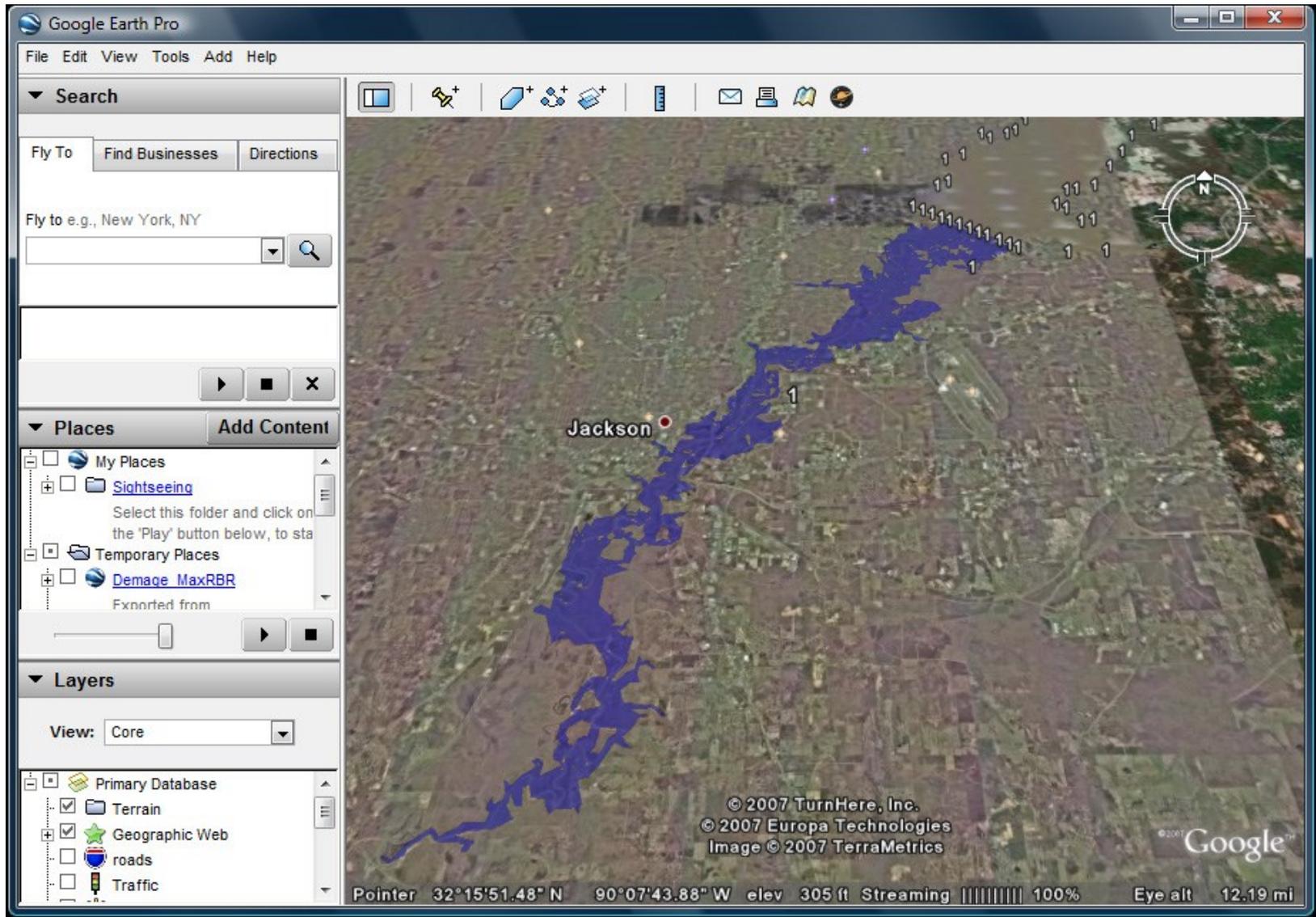
Perdite umane



Mappa dei danni



# Decision Support System

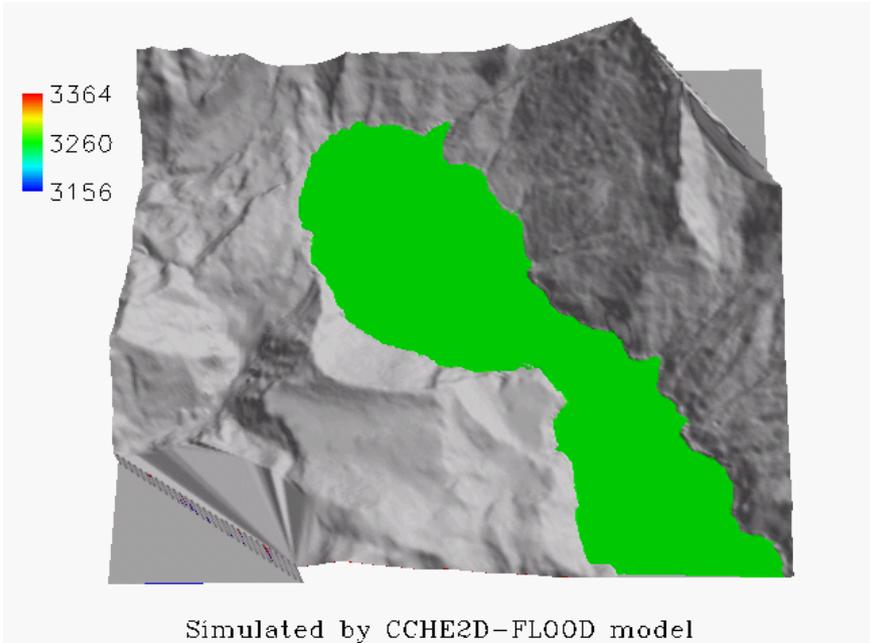




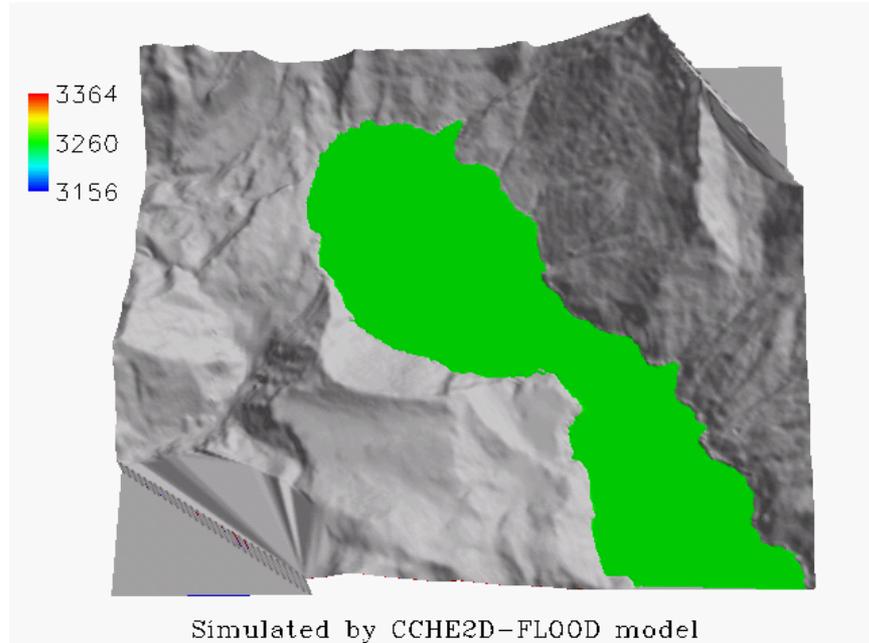
# Esondazioni e inondazioni dovuti a frane



# Esondazioni e inondazioni dovuti a frane



Velocità della frana 14m/s

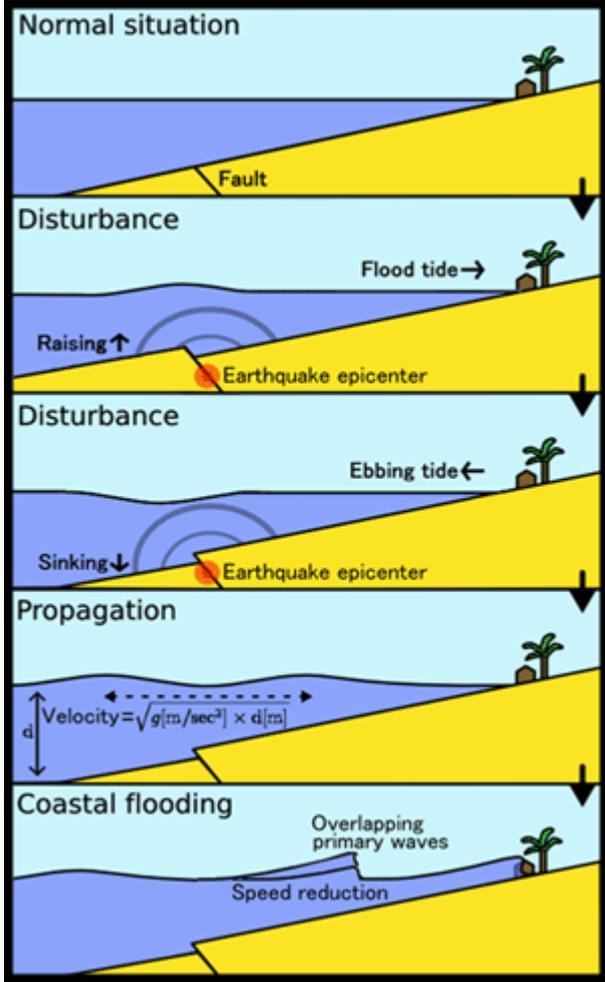


Velocità della frana 20m/s

GPU



# Tsunami



1. [Sumatra](#)
2. [Giappone](#)



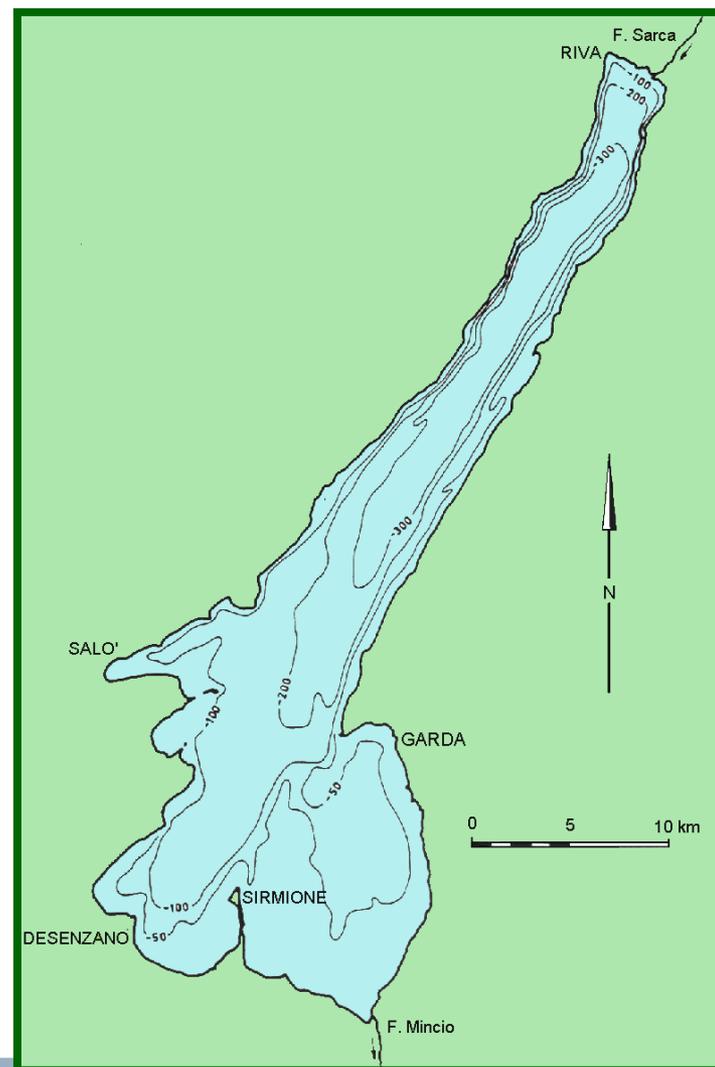
## Il Lago di Garda: alcuni dati

Il Lago di Garda è la maggior riserva d'acqua dolce italiana.

Misure:

- area: 379 km<sup>2</sup>,
- perimetro: 185 km,
- altitudine: 65 m s.l.m.
- profondità massima: 346 m,
- profondità media: 133 m,
- asse maggiore: 51.6 km,
- asse minore: 17.2 km,
- portata media dell'emissario:  
58 m<sup>3</sup>/s

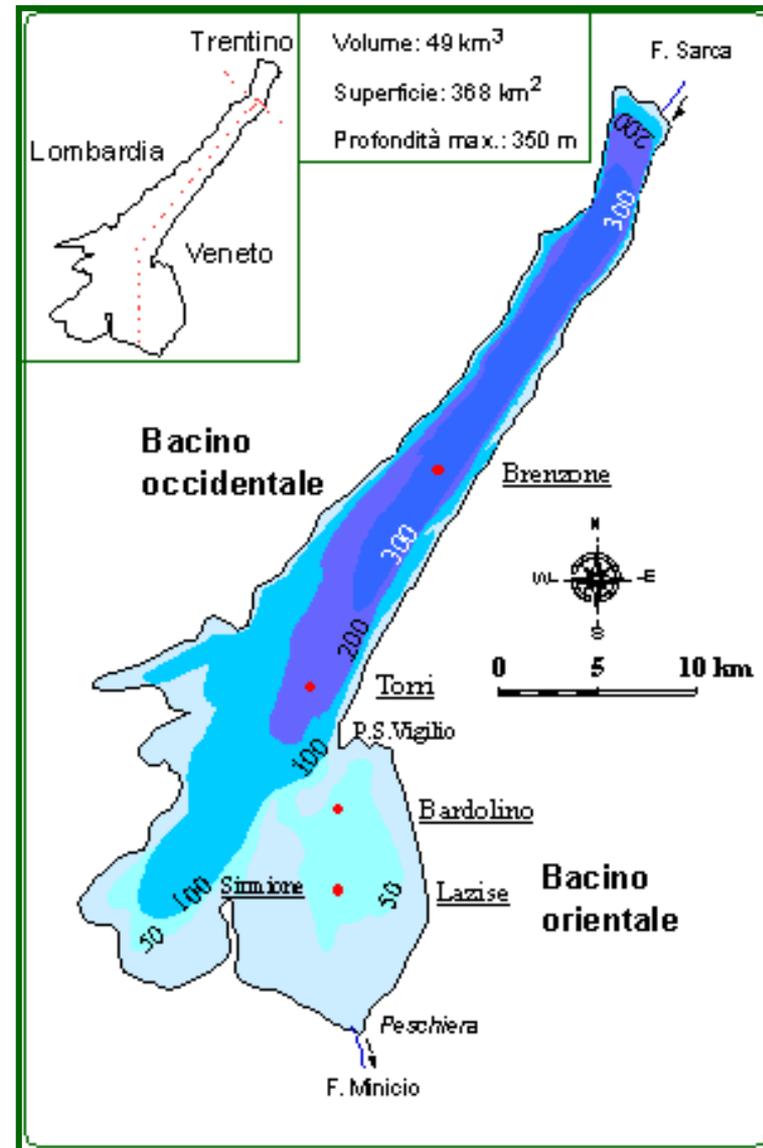
La circolazione è largamente influenzata dagli effetti del vento





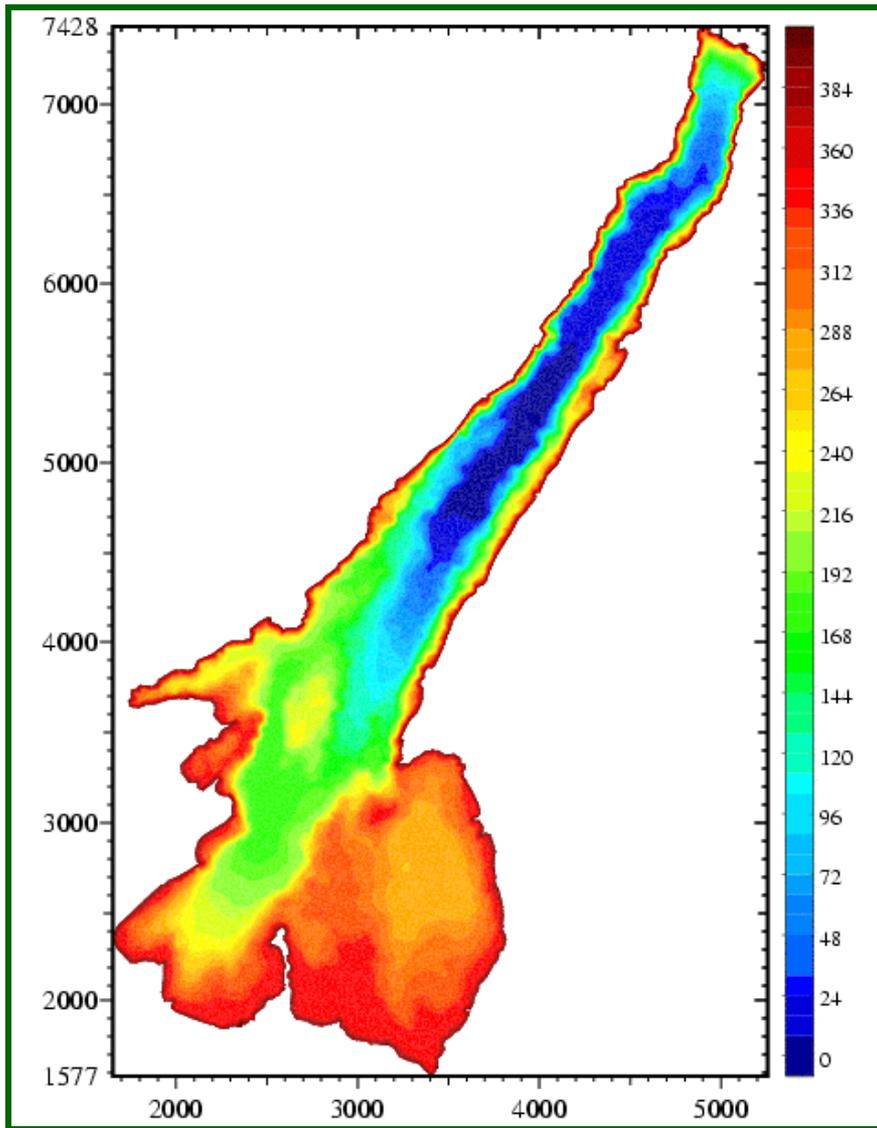
Sulla base della batimetria può essere diviso in due bacini, orientale ed occidentale, divisi da una dorsale sommersa dall'andamento sinuoso che congiunge la penisola di Sirmione con Punta San Vigilio.

In corrispondenza della Secca del Vo, posta circa 3 km a sud di P.S. Vigilio, ed integrata nella dorsale, le profondità sono inferiori a 10 m.

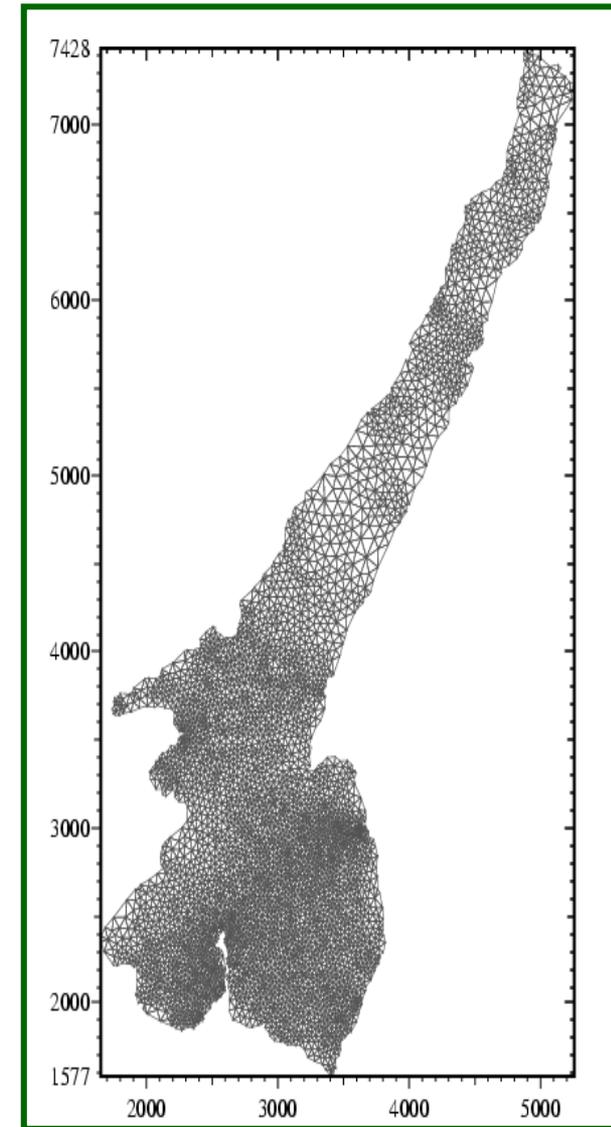




# La griglia di calcolo



Circa 500000  
incognite.





**24 novembre 2004, ore 23.59: si e' registrato un forte evento sismico di magnitudo 5.4, localizzato nella riva sinistra del Lago di Garda, al confine tra le province di Brescia, Trento e Verona.**

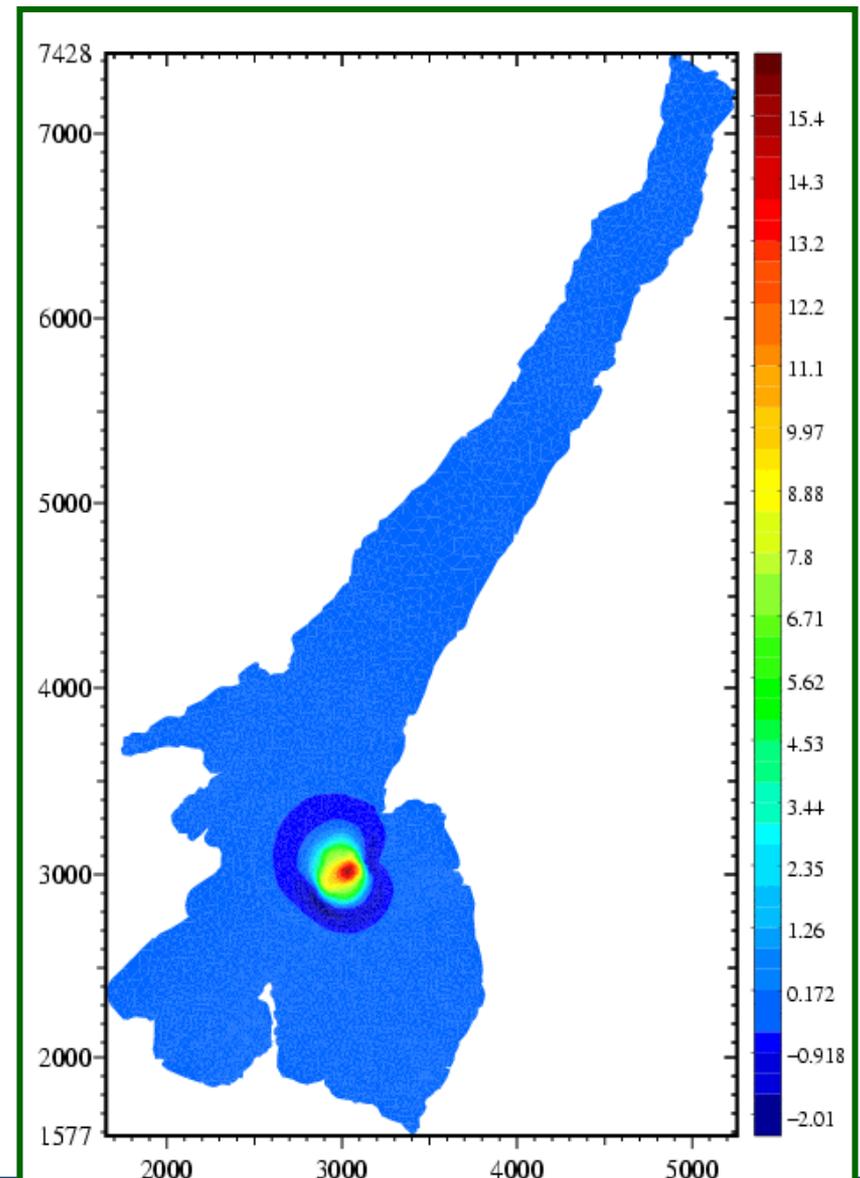
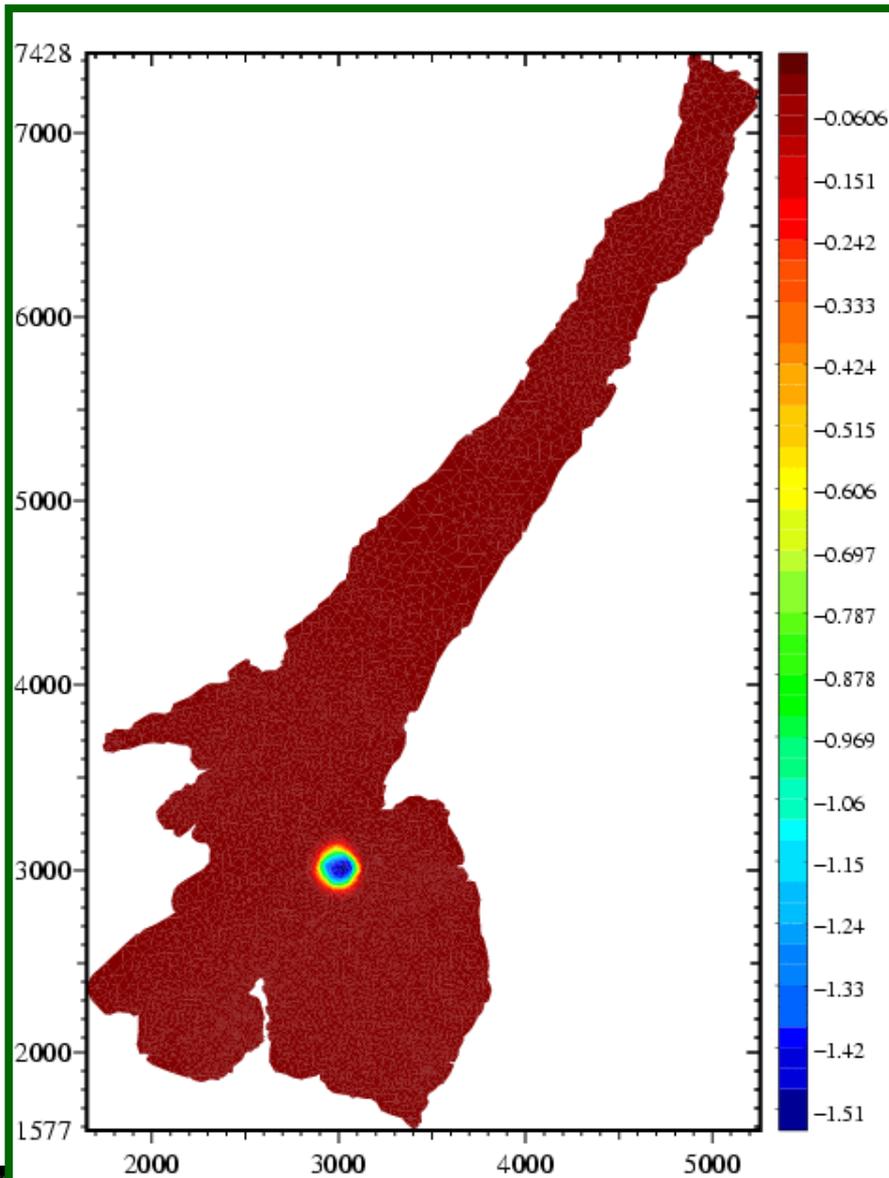


**Salò: danni ad edifici**

**Supponiamo un movimento ondulatorio di 0.5 secondi con ampiezza massima di 2m con epicentro di fronte a punta S.Vigilio.**



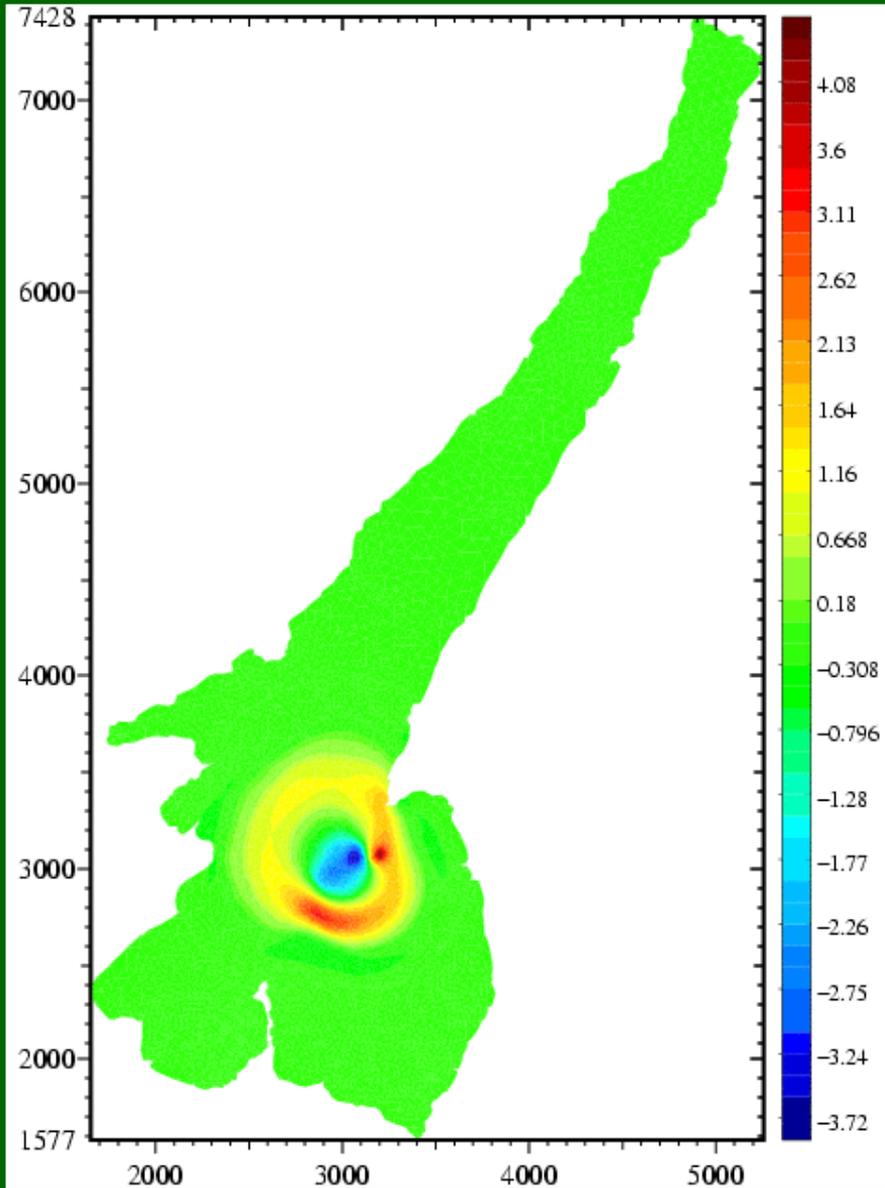
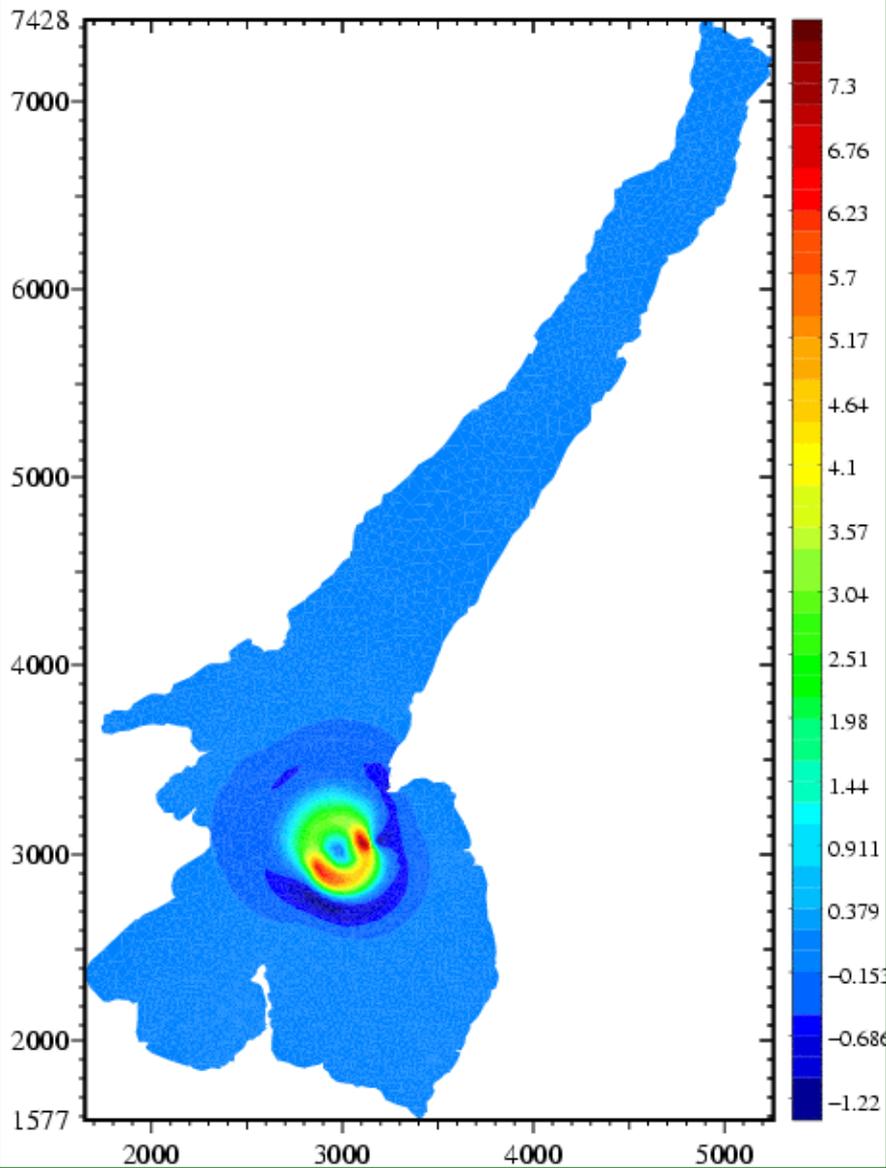
# L'onda anomala prodotta da un movimento sul fondo





20'

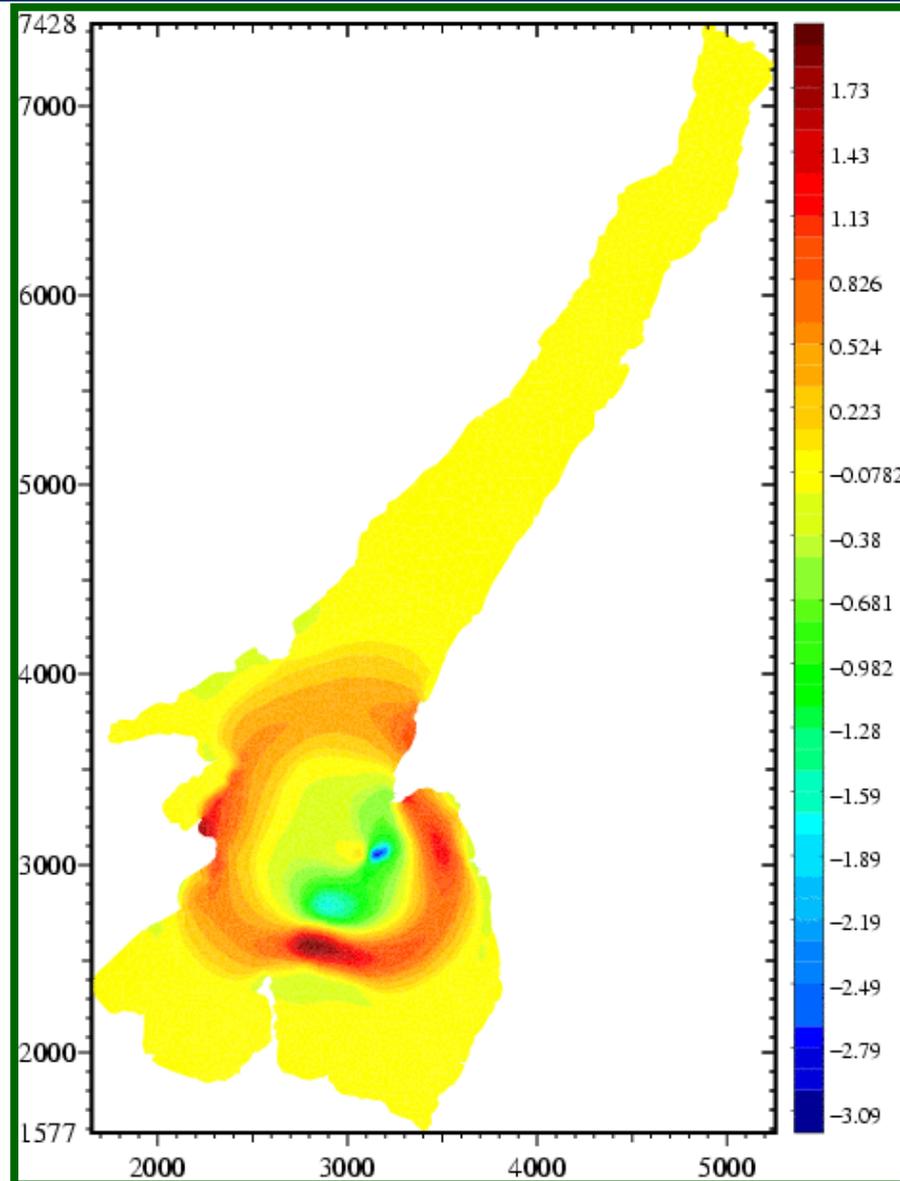
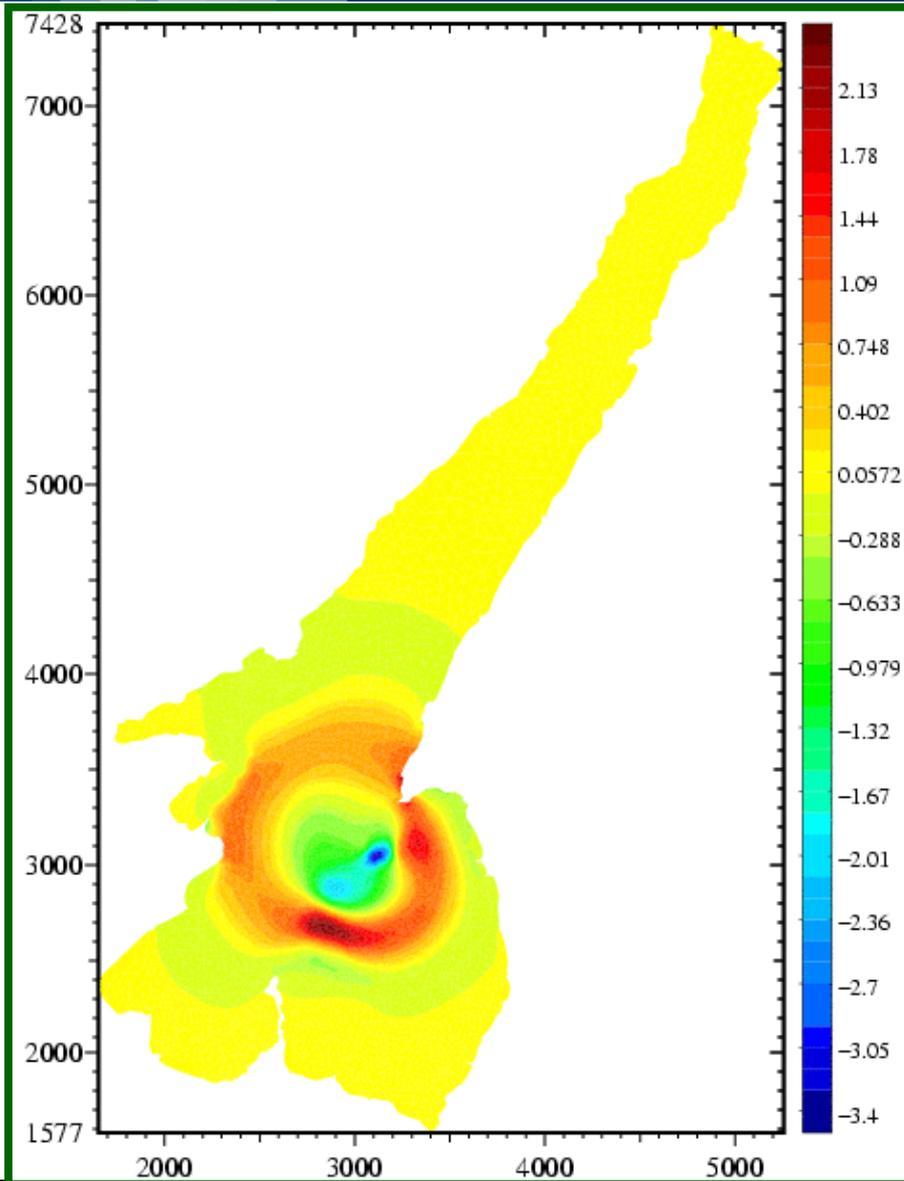
40'





60'

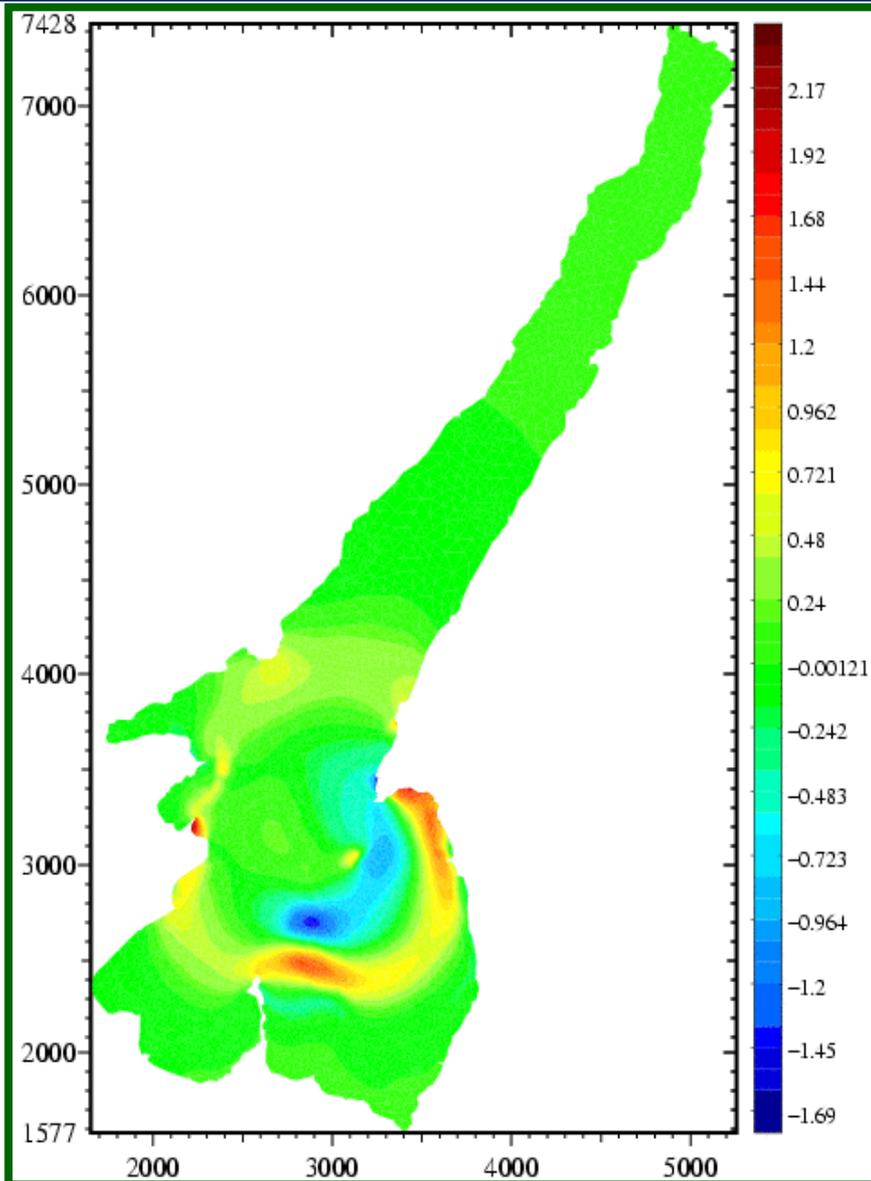
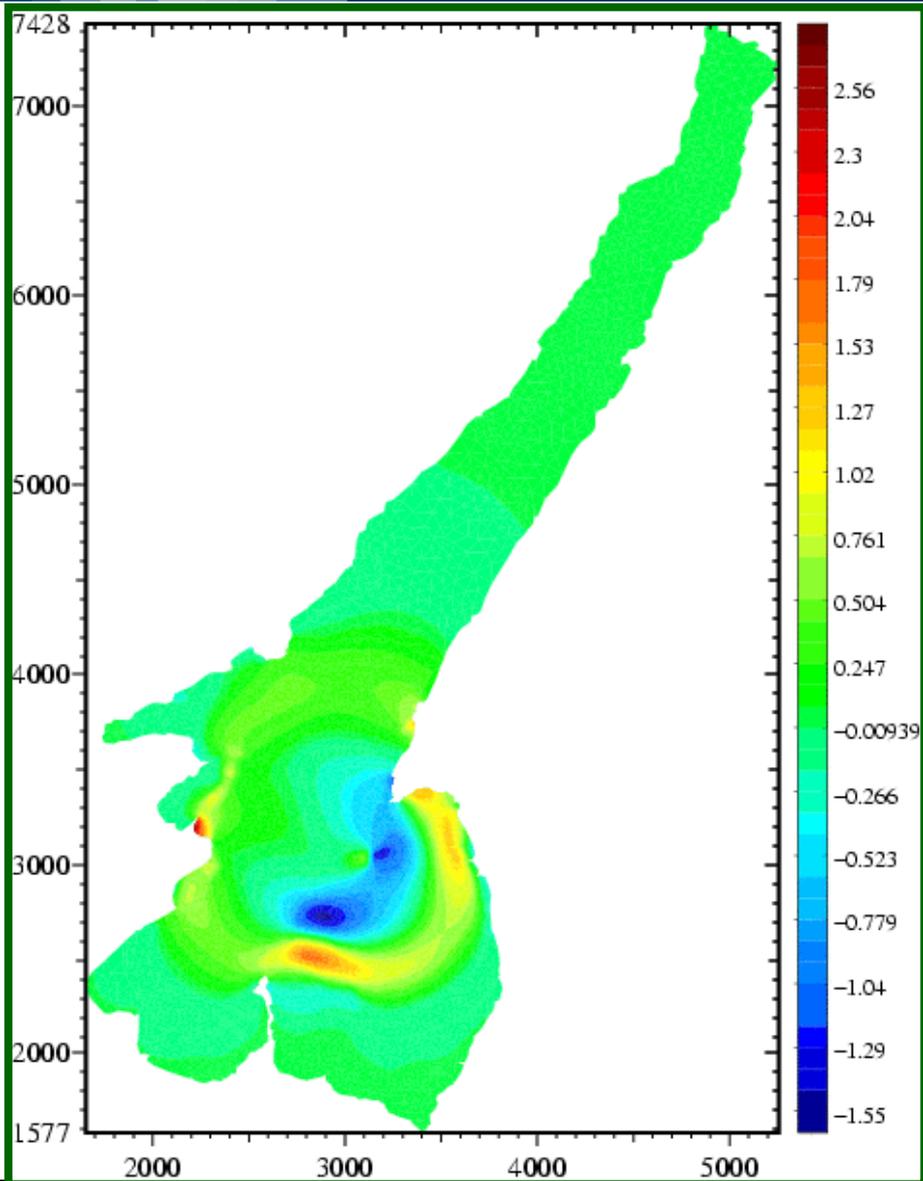
80'





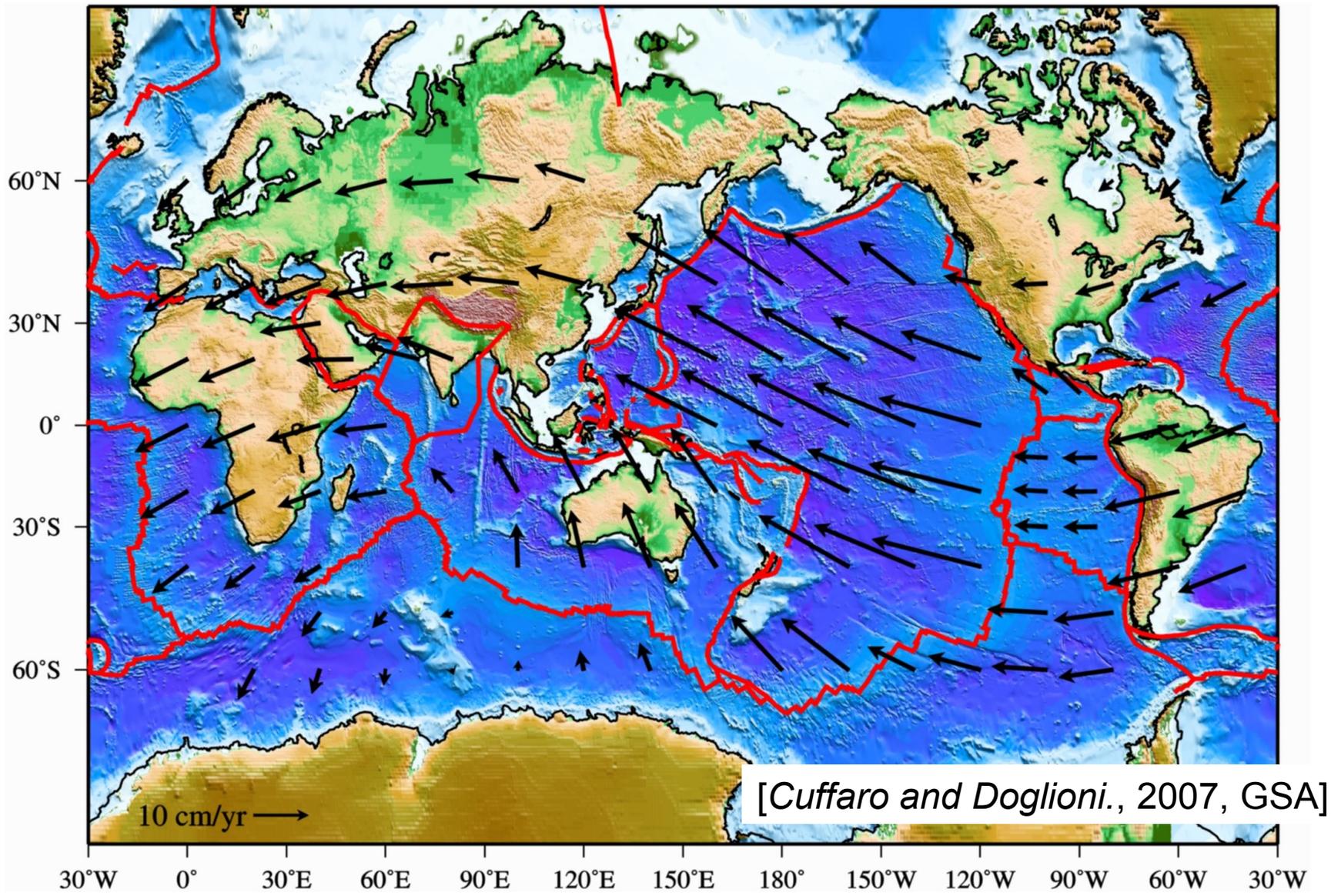
100'

120'





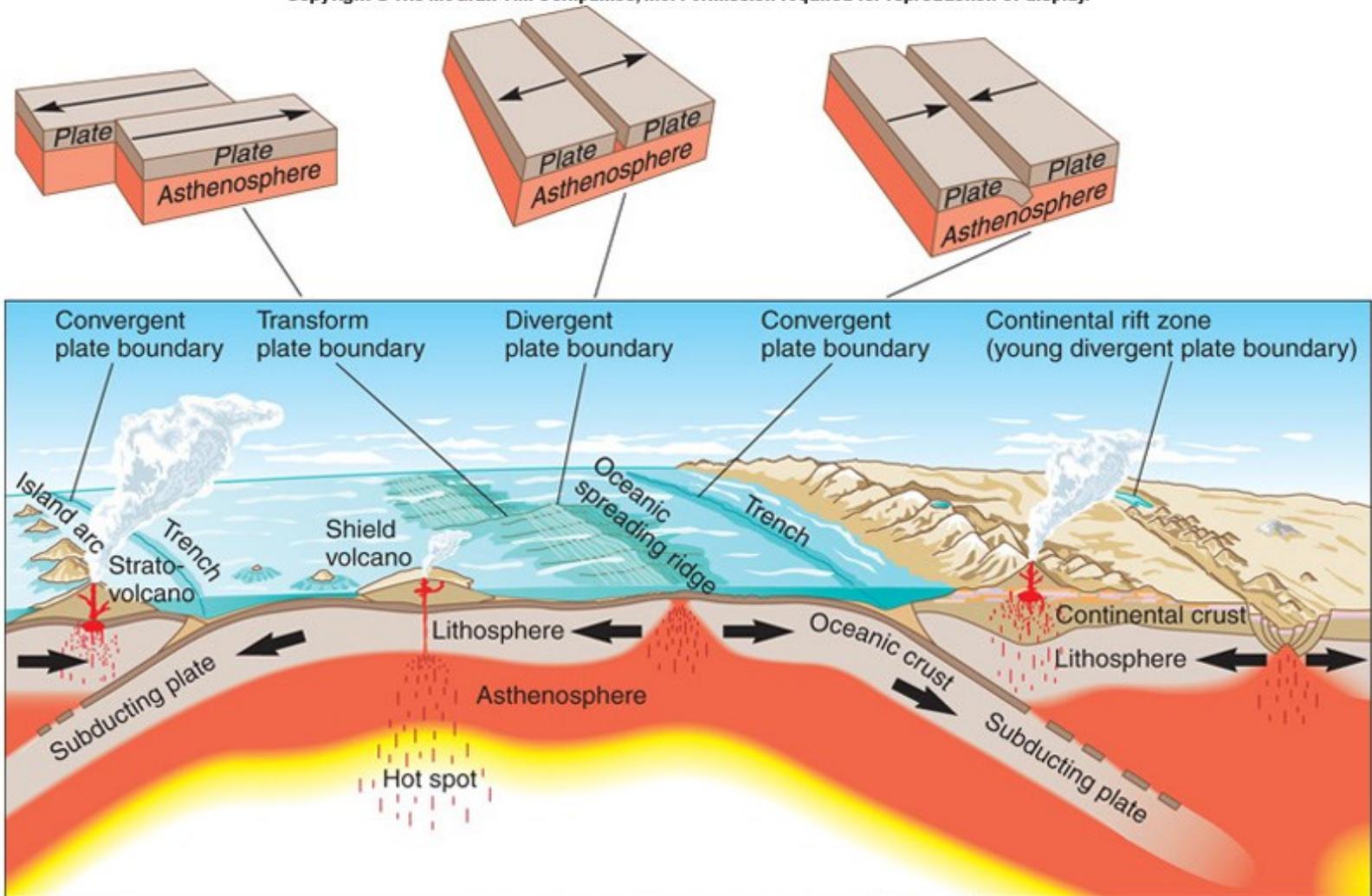
# Hot-spot frame (2)





# Plate boundaries (1)

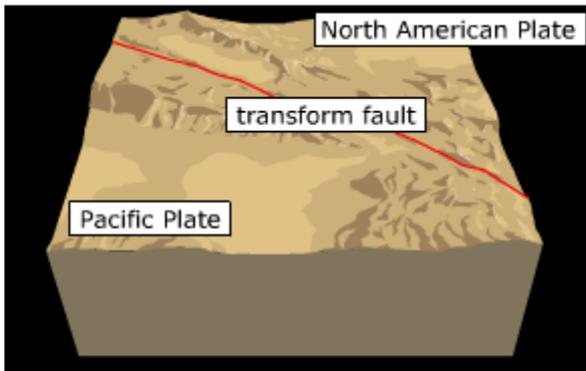
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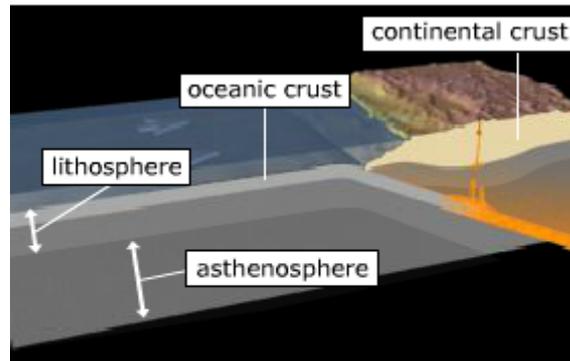
Kious and Tilling, US Geological Survey.



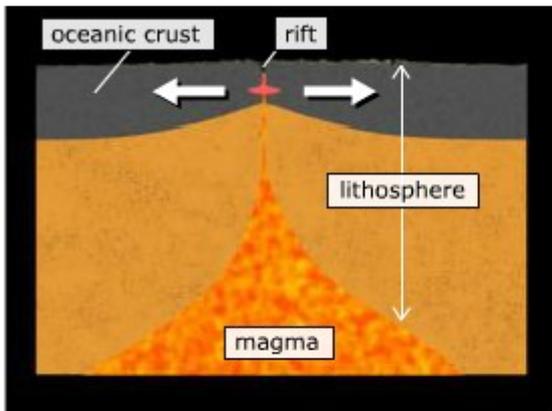
# Plate boundaries (2)



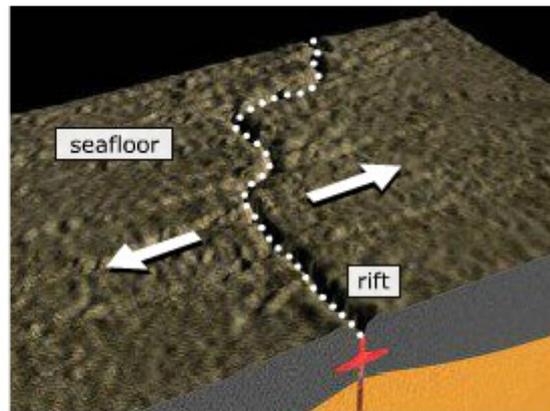
Transform Boundary



Convergent Boundary



Divergent Boundary





# Processi di subduzione

$$\begin{cases} \operatorname{div} \boldsymbol{\sigma} + \rho_l \mathbf{g} = 0 & \text{in } \Omega_t \\ \frac{D\boldsymbol{\sigma}}{Dt} = \mathcal{M}(\boldsymbol{\sigma}, \mathbf{d}) & \text{in } \Omega_t \end{cases} + b.c.$$

$\boldsymbol{\sigma}$  = stress tensor

$\mathbf{d}$  = strain rate tensor

$\mathcal{M}$  = constitutive law

$\mathbf{v}$  = displacement

$\mu$  = Coulomb coeff.

$\delta v_n \leq 0, \quad \sigma_n \leq 0, \quad \delta v_n \sigma_n = 0,$  Signorini conditions

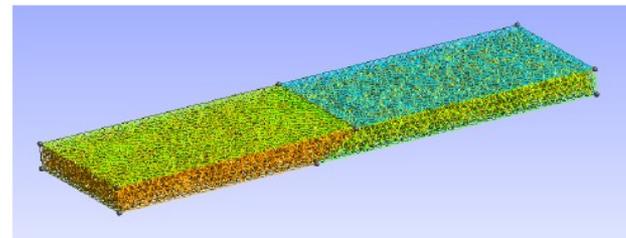
$$\left\{ \begin{array}{l} \|\sigma_t\| \leq -\mu\sigma_n \quad \text{if } \delta v_t = 0, \\ \sigma_t = \mu\sigma_n \frac{\delta v_t}{\|\delta v_t\|} \quad \text{if } \delta v_t \neq 0, \end{array} \right.$$

Coulomb friction



## Parametri della simulazione

Physical quantity	Value
Length	1000 km
Width	600 km
Thickness	100 km
Subduction angle	30°
Young modulus	$10^{11}$ Pa
Poisson coeff.	0.25
Cohesion	$10^7$ Pa
Internal friction angle	15°
Velocity of the fast plate	7 cm/a
Velocity of the slow plate	2 cm/a



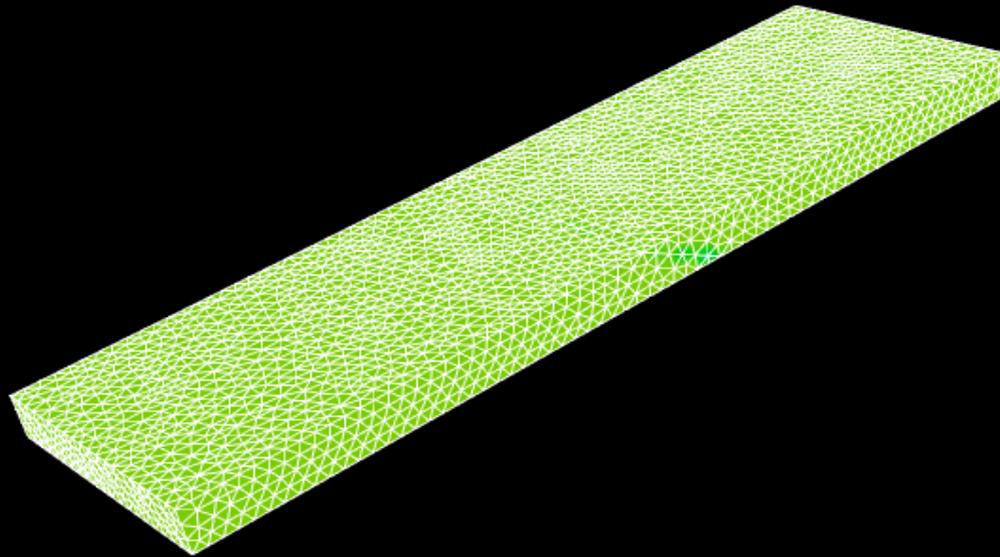
Numerical parameter	Value
Number of nodes	7620
Number of elements	33064
Time step	530 a
Number of time step	30000



# Subduction - Moving plates (W-drift) with friction



Z- Velocity component (m/s)





**2008 – 2009**

“Studio matematico sulle esondazioni del fiume Po”

Autori: Labò, Nguyen, Zanaboni

Tutor universitario: Edie Miglio

Docenti: Busconi, Colombini, Dallanocce

Scuola: Liceo Scientifico “L. Respighi” – Piacenza

Partecipazione al Workshop Isef-Intel (San Josè, CA - USA, 9-15 maggio 2010)



Bet on math <http://betonmath.polimi.it/>



Ricerchiamo metodologie didattiche di prevenzione e sensibilizzazione,  
basate sull'insegnamento della probabilità



GRAZIE PER L'ATTENZIONE

